Newton-conjugate-gradient methods for computations of solitary waves and their linear-stability eigenvalues

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Abstract:

Computations of solitary waves and their linear-stability eigenvalues are important issues in nonlinear waves. So far, a number of iteration methods have been developed for these computations. Examples include the Petvisahvili method [1], the squared-operator iteration method [2], and the accelerated imaginary-time evolution method [3] for solitary-wave computations, as well as their counterparts for eigenvalue computations [4]. However, convergence of these methods can still be slow in many situations. In this talk, we propose Newton-conjugate-gradient methods for these computations. These methods are based on Newton iterations, coupled with conjugate-gradient iterations to solve the resulting linear Newton-correction equation. For solitary waves, the preconditioned conjugate-gradient method is proposed to directly solve the underlying Newton-correction equation. For linear-stability eigenvalues, the underlying Newton-correction equation is first turned into a positive-definite normal equation, which is then solved by the preconditioned conjugate-gradient method. These methods will be applied to compute both the ground states and excited states of solitary waves, as well as their stable and unstable discrete linear-stability eigenvalues, for a large number of physical problems such as the two-dimensional NLS equations with and without periodic potentials in optics, and the fifth-order KP equation in water waves. Numerical results show that these proposed methods are *much* faster than the other existing numerical methods, often by orders of magnitude. In addition, these methods are very robust and always converge in all the examples being tested. Furthermore, they are very easy to implement. This talk is based in part on the materials in [5, 6].

References:

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